

~~External~~ Extended surface "Fins"

$Q_{conv} = h * A_s * (T_s - T_{\infty}) \rightarrow \Delta T \approx \text{constant}$ according to the applications.

To increase " A_s " using an extended surface (Fins) which protrudes from the surface exposed to heat transfer.

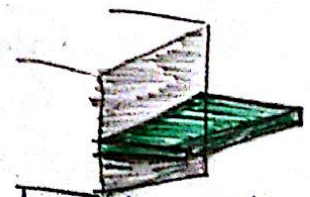
To increase " h " $\rightarrow$ increase the moving fluid velocity by using an external means to forcing the fluid to move.

External means $\xrightarrow{\text{such as}}$ Fan, blower, compressor [Gases]

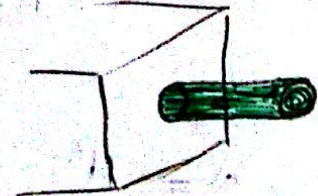
Pump $\rightarrow$ [liquids]



(Annular Fin)



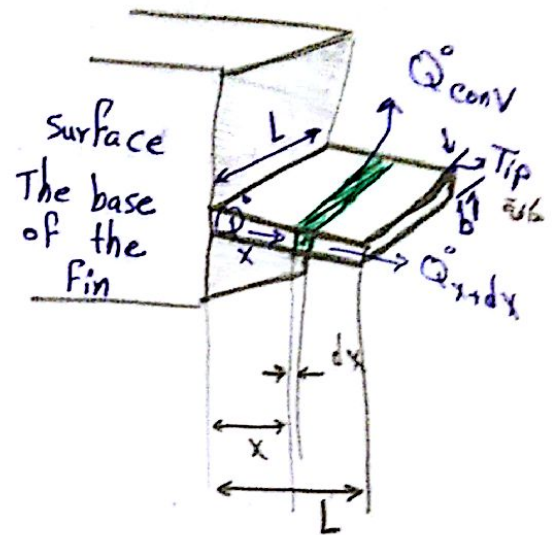
(rectangular Fin)



circular (Pipe) Fin.

Assumptions

- 1 - $h = \text{const}$
- $K = \text{const}$
- $A_c = \text{const}$
- $P = \text{const}$



Applying heat balance on the element shown where

$$Q_{in} = Q_{out}$$

$$Q_x = Q_{x+dx} + Q_{conv}$$

where $Q_{x+dx} = Q_x + \frac{\partial}{\partial x} (Q_x) dx$

$$= Q_x + \frac{\partial}{\partial x} \left(-K * A_c * \frac{dT}{dx} \right) \cdot dx$$

$$Q_{conv} = h * (P \cdot dx) * (T - T_{\infty})$$

$$\cancel{Q_x} = \cancel{Q_x} + \frac{d}{dx} \left[-K A_c \frac{dT}{dx} \right] \cdot dx + h (P \cdot dx) (T - T_{\infty})$$

$$0 = -K A_c \frac{d^2 T}{dx^2} \cdot dx + h (P \cdot dx) (T - T_{\infty})$$

$$0 = \frac{d^2 T}{dx^2} - \underbrace{\frac{hP}{KA_c}}_{m^2} (T - T_{\infty}) \rightarrow \frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\frac{dT^2}{dx^2} = \frac{d\theta^2}{dx^2}$$

$$\frac{dx}{dx} \div$$

$$-KA_c \div$$

$0 = \frac{d^2 \theta}{dx^2} - m^2 \theta \rightarrow$ Homogenous differentiation equation
 with constant coefficient. $\downarrow \frac{1}{e^{mx}}$

$$0 = D^2 - m^2$$

. $\Rightarrow$ $D = \pm m$

$\therefore e^{mx} \text{ and } e^{-mx}$

$$\theta(x) = e_1 \cdot e^{mx} + e_2 e^{-mx}$$

D-operator
 $\frac{d^2 \theta}{dx^2} = D^2$
 $\frac{d\theta}{dx} = D$
 $\theta = 1$

* Very long (Infinite) fin ($L \rightarrow \infty$)

$$\theta_{\text{tip}} = T_{\text{Tip}} - T_{\infty} = 0$$

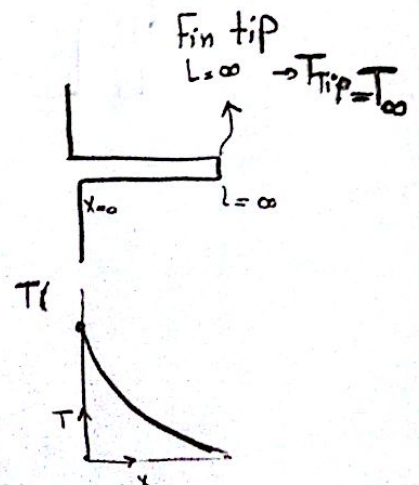
B.c at $x=0 \rightarrow T=T_0$ ($\theta = \theta_0$)

at ($x=L \rightarrow \infty$) ($\theta_{\text{tip}} = 0$)

بالتعويض $\rightarrow \theta(x) = e_1 \cdot e^{mx} + e_2 e^{-mx} \rightarrow I$

$$\therefore \theta_0 = e_1 + e_2 \rightarrow (2)$$

$$0 = e_1 e^{\infty} + e_2 * e^{-\infty}$$



d)

$$0 = C_1 * e^{\infty}$$

$$0 = C_1 * \infty$$

$$\therefore C_1 = 0$$

بالنوعين
2)

$$\Theta_0 = C_2$$

$$\therefore \boxed{\Theta(x) = \Theta_0 \cdot e^{-mx}}$$

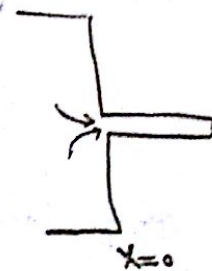
considering conduction at fin base

$$\text{at } x=0 \rightarrow Q = -K \cdot A_c \left. \frac{d\Theta}{dx} \right|_{x=0}$$

$$d\Theta_x = \Theta_0 e^{-mx}$$

$$\frac{d\Theta}{dx} = \Theta_0 e^{-mx} * -m$$

$$\left. \frac{d\Theta}{dx} \right|_{x=0} = -m \Theta_0$$



$$\therefore Q_{fin}^{\circ} = Q_{cond}^{\circ} \Big|_{x=0} = -K * A_c * -m * \Theta_0$$

$$= +K A_c * \sqrt{\frac{hP}{K A_c}} * \Theta_0$$

$$\boxed{Q_{fin}^{\circ} = \sqrt{hP K A_c} * \Theta_0}$$

very long $(T - T_{\infty})$

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بالحدود
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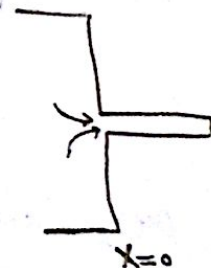
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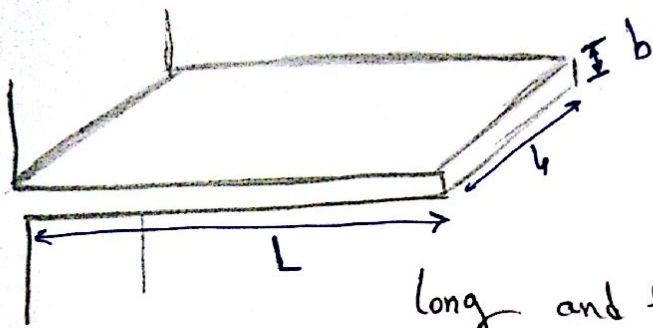


$$\therefore Q_{fin}^{\circ} = Q_{cond}^{\circ} \Big|_{x=0} = -K * A_c * -m * \Theta_0$$

$$= +K A_c * \sqrt{\frac{hP}{K A_c}} * \Theta_0$$

$$Q_{fin}^{\circ} = \sqrt{hP K A_c} * \Theta_0$$

very long $(T - T_{\infty})$



المعادلة العامة

e

Long and thin fin = Insulated tip fin

Negligible heat transfer at fin end (tip).

at $x=0 \rightarrow T=T_0 \rightarrow \theta = \theta_0$

at $x=L \rightarrow \dot{Q}=0 \Rightarrow \boxed{\frac{d\theta}{dx} \big|_{x=L} = 0}$

$$\theta(x) = \theta_0 \cdot \frac{\cosh m(L-x)}{\cosh mL}$$

$$\dot{Q}_{fin} = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_0 \cdot \tanh mL$$

short fin with convective end

المعادلة العامة

B.c at $x=0 \rightarrow T=T_0 = T_b \rightarrow \theta$

at $x=L \rightarrow \dot{Q}_{cond} = \dot{Q}_{conv}$

$$-k A_c \frac{d\theta}{dx} = h A_c \cdot \theta$$

$$\theta(x) = \theta_0 \cdot \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$$

$$\dot{Q}_{fin} = \sqrt{h P k A_c} \cdot \theta_b \cdot \frac{\sinh mL + \left(\frac{h}{mk}\right) \cosh mL}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$$